2. Electrostatic Transducers

2.1 Fundamentals of electrostatic transducers

The electrostatic (or capacitive) conversion is based on the interaction between the mechanical and electrical quantities in a condenser having one electrode fixed and the other electrode movable. The displacement of the movable electrode can be caused either by an external force in the case of a sensor or by a voltage in the case of an actuator. Based on the displacement orientation of the movable electrode with respect to the fixed electrode, we can distinguish two basic types of capacitive transducers: transversal and lateral (see Figure 2.1).

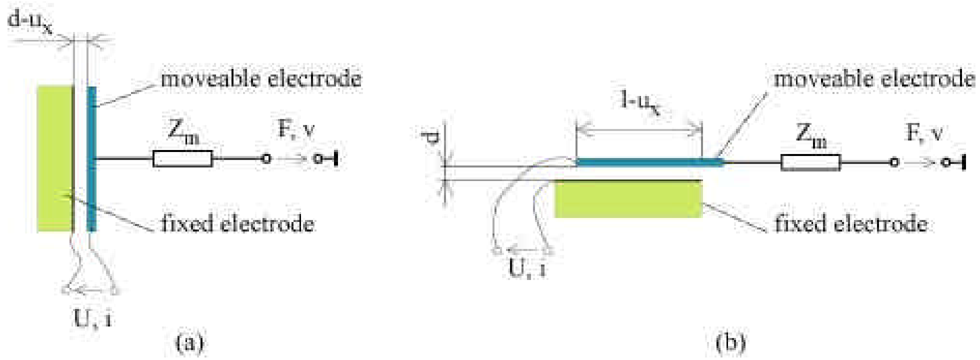

Figure 2.1 (a) Transversal, (b) lateral electrostatic transducer.

In a **transversal capacitive transducer**, the movable electrode is displaced off-plane, perpendicularly to the electrode surface (see Figure 2.1 a). The original distance between the electrodes d varies by the displacement u_x when the transducer is in function. The instantaneous transducer capacity is:

$$C = \frac{eA}{d - u_x} \tag{1}$$

where e is the permittivity of the dielectric between the electrodes and A is the electrode surface area. When the voltage U (or the charge Q) is applied between the electrodes, the energy of the system can be expressed as:

$$W = \frac{CU^2}{2} = \frac{Q^2}{2C} = \frac{eAU^2}{2(d - u_x)} = \frac{(d - u_x)Q^2}{2Ae} \tag{2}$$

Using the principle of virtual work, the differentiation of the Equation (2) relating to the displacement u_x and to the charge Q (or to the voltage U) gives the relations for the mechanical and electrical quantities. We can thus obtain:

$$dW = \left.\frac{\partial W}{\partial u_x}\right|_U du_x + \left.\frac{\partial W}{\partial U}\right|_{u_x} dU = F du_x + Q dU \tag{3}$$

Supposing that the voltage doesn't change during the movement of the movable electrode, we obtain the following expression for the force:

$$F_U = \frac{eAU^2}{2(d - u_x)^2} \tag{4}$$

Supposing that the charge doesn't change during the movement of the movable electrode, we obtain, by a similar operation, the force as:

$$F_Q = \frac{Q^2}{2eA} \tag{5}$$

In practice, capacitive transducers operate between two regimes: with a constant voltage or with a constant charge. In these two cases, the quadratic relation between the force and the voltage or the charge can be linearized by the application of the polarization voltage or the polarization charge on which the alternating quantities are superposed.

If we consider small signals, we can express mechanical and electrical quantities of the transducer in a frequency domain. We thus have for a capacitive transducer with a constant voltage:

$$I = jwC_0U + NV \quad , \quad F = NU + \frac{1}{jwc_n}V \tag{6}$$

where $N = \frac{Q_0}{d_0}$ is a transduction factor, Q_0 is the static charge, $d_0 = d - u_0$ is the distance between the electrodes at rest, $c_n = -\frac{C_0}{N^2}$ is the negative compliance, C_0 is the capacity at rest, I is the current passing through the condenser, and V is the velocity of the movable electrode. The two Equations (6) can be represented by the analogue circuit shown in Figure 2.2.

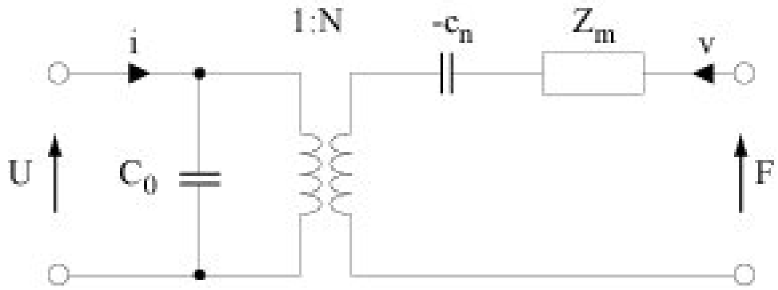

Figure 2.2 Analogue circuit of electrostatic transducer.

The impedance Z_m has been added to this circuit in order to represent the effect of mechanical components attached to the transducer. The presence of the negative compliance is related to the transducer stability. In order to provide a stable operation of the transducer (without collapsing the movable electrode towards the fixed one), the external compliance c must be added to the system. This compliance that is a part of the impedance Z_m must satisfy the condition:

$$c < |c_n| \tag{7}$$

This condition can be expressed by the following relation for the value of the polarization voltage:

$$U_0 < \sqrt{\frac{8d^3}{27eAc}} \tag{8}$$

The limiting value of the polarization voltage U_0 is called **pull-in voltage**. If the transducer works in the mode with a constant charge, the negative compliance does not appear.

The transducer polarization that is used in order to linearize its functioning may be realized by a dielectric material, called electret, which conserves permanently the electric polarisation.

The application of an electret can be realized by a deposition of an electret layer either on the movable or on the fixed electrode [SES 80].

In a **lateral capacitive transducer**, the movable electrode is displaced in-plane, in parallel to the electrode surface (see Figure 2.1 b). During the transducer operation, the distance between the electrodes doesn't change; the common area of the electrodes varies. When considering rectangular electrodes of the width *h* and of the length *l*, moving in parallel to their length, the instantaneous capacity of the transducer is:

$$C = \frac{e\,h(l-u_x)}{d} \tag{9}$$

Using the same reasoning as in the case of a transversal electrostatic transducer, we obtain equations for electrical and mechanical quantities for the lateral form of transducer. For both modes of operation, which are constant voltage mode and constant charge mode, we obtain the following equations:

$$I = jwC_0U + NV \quad , \quad F = NU \tag{10}$$

where $N = \frac{Q_0}{l_0}$ is the conversion factor and $l_0 = l - u_0$ is a common part of length of electrodes in a rest position. These two equations can be represented by a similar analogue circuit as in the case of a transversal electrostatic transducer (see Figure 2.2). The only difference is that the negative compliance is not present in the case of the lateral electrostatic transducer.

The two forms of electrostatic transducer, transversal and lateral, can be applied in an interdigital arrangement as shown in Figure 2.3. These configurations are very often used in the field of microsystems due to their ability to be easily fabricated by micromachining.

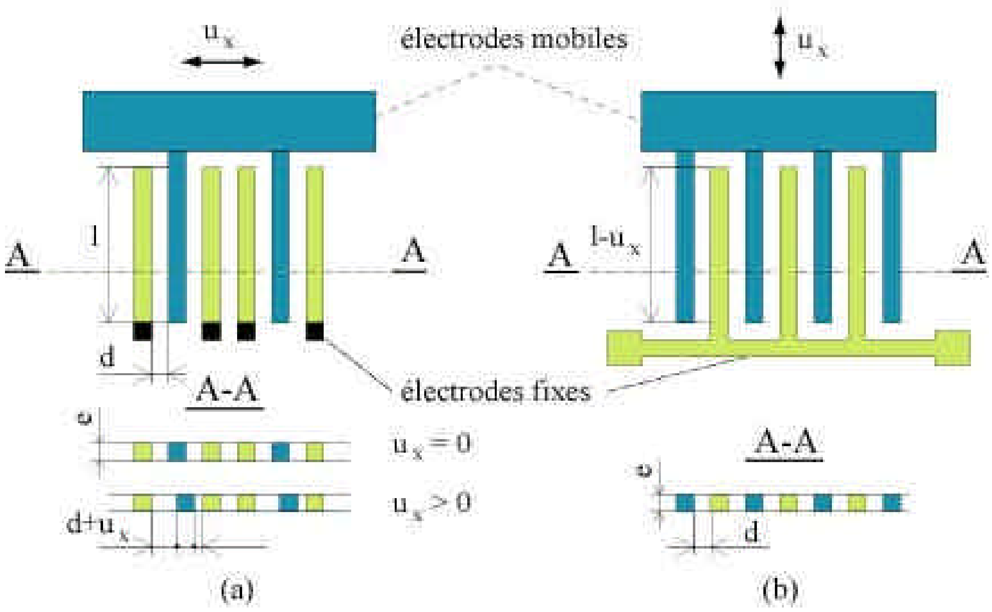

Figure 2.3 (a) Transversal, (b) lateral interdigital electrostatic transducer.

In an **interdigitated transducer of a transversal form** (see Figure 2.3 a), *n* electrodes are displaced perpendicularly to their length. The total inter-electrode capacitance C_T is composed of two parts, C_{T1}, C_{T2}, linked in a serial differential configuration. If we do not consider parasitic capacitances, the capacitances C_{T1} and C_{T2}, can be expressed as follows:

$$C_{T1} = n\frac{ele}{d+u_x} \quad , \quad C_{T2} = n\frac{ele}{d-u_x} \tag{11}$$

The total capacitance at rest will be:

$$C_0 = 2n\frac{e\,l\,e}{d} \tag{12}$$

The conversion factor for a transversal transducer form is $N = \frac{Q_0}{d}$.

In an **interdigitated transducer of a lateral form** (see Figure 2.3 b), *n* electrodes are displaced in parallel to their length. The total inter-electrode capacitance C_L is:

$$C_L = n\frac{ee(l-u_x)}{d} \tag{13}$$

The conversion factor for this form of transducer is $N = \frac{Q_0}{l}$. More details about an interdigitated capacitive transducer can be found in [JOH 95].

Another approach in building a transversal capacitive transducer is described in [LEG 97].

This transducer consists of a laterally compliant cantilever beam and a fixed curved electrode; both suspended above a ground plane (see Figure 2.4). The device was fabricated by polysilicon surface micromachining techniques.

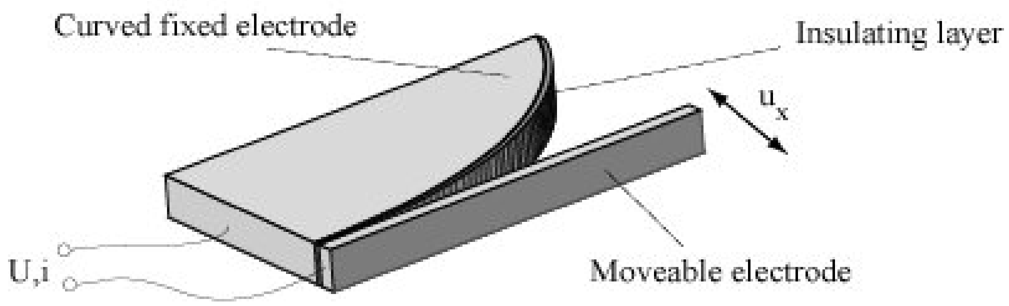

Figure 2.4 Electrostatic transducer with curved electrode.

Because of the possibility to generate relatively large displacements and high force, this transducer type is well matched for the use as an actuator. From the schematics shown in Figure 2.4, we can see that the gap distance between electrodes is small near the clamped edge of the beam and increases with the position along the length of the beam. When a voltage is applied across the gap, an electrostatic force is created that deforms the beam along the outline of the curved electrode. The displacement is parallel to the wafer surface. In this way, the shape of the curved electrode can be easily adjusted by changing the mask design. To prevent a short circuit between the beam and the curved electrodes, electrical insulation is required.

2.2 Related Reading

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