1. Piezoresistive Transducers

1.1 Fundamentals of piezoresistive transducers

Piezoresistivity is the dependence of electrical resistivity on strain and consequently on stress. Strains change the internal atom positions and their motions and hence, the resistivity. First strain gauges based on the piezoresistive effect consisting of metal wires on a paper carrier are still in use in different applications. Later, it was demonstrated that semiconductors show a much greater piezoresistive effect and since that time, silicon based strain gauges were rapidly developing.

We can explain the basic resistivity relationships if we take as an example the case of a long solid bar as shown in Fig.1.1.

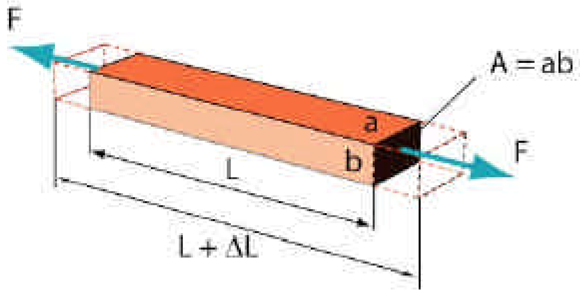

Figure 1.1 Bar dimensions used for resistivity definition.

When a load, F is applied, the object deforms, causing the resistance change. For the resistance, R of this bar we can write:

$$R = \frac{L}{A}\rho \tag{1}$$

where L is the length, A is the cross-section and ρ is the resistivity of the material. Differentiating the preceding equation, we obtain:

$$\frac{dR}{R} = \frac{dL}{L} - \frac{dA}{A} + \frac{d\rho}{\rho} \tag{2}$$

We can see from the preceding expression that there are two components of the piezoresistive effect – the geometric component and the resistive component. The geometric component can be written with the aid of Poisson ratio, ν that is defined as the relative increase in the length S_L to the relative decrease in the diameter S_D as:

$$\upsilon = -\frac{S_L}{S_D} = -\frac{da/a}{dL/L} = -\frac{db/b}{dL/L} \tag{3}$$

The relative resistance change can be then written as:

$$\frac{dR}{R} = \frac{dL}{L}(1+2\upsilon) + \frac{d\rho}{\rho} \tag{4}$$

It is convenient to define the gauge factor K as:

$$K = \frac{dR/R}{dL/L} = 1 + 2\upsilon + \frac{d\rho/\rho}{dL/L} \tag{5}$$

In metals, the gauge factor is defined mostly by the geometric component, which is expressed by two first members of the preceding expression. For this reason, the gauge factors of metals are rather small and attain the values between 2 and 4. It was shown that gauge factor of semiconductors is much higher and that doped silicon exhibits a gauge factor which can be as large as 200, depending on the amount of doping. This effect is the result of the resistive component corresponding to the third member of the preceding expression.

To formulate the resistive component of piezoresistivity relations in a material, we need to define the resistivity in an anisotropic material as a relation of the three-directional components of the electric field, **E** to the three-directional components of current flow, **J**. This relation is made via resistivity, r defined as a second-rank tensor that has nine elements expressed in a 3×3 matrix. These elements reduce to six independent values from symmetry considerations:

$$\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} \rho_1 & \rho_6 & \rho_5 \\ \rho_6 & \rho_2 & \rho_4 \\ \rho_5 & \rho_4 & \rho_3 \end{bmatrix} \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} \tag{6}$$

In the case of silicon crystal that has a cubic lattice structure, if the Cartesian axes are aligned to the (100) axes, the entries $?_1$, $?_2$ and $?_3$ will be equal, as they all represent resistance along the (100) axes and are denoted by ?. The remaining components of the resistivity matrix, which represent cross-axis resistivities, will be zero because unstressed silicon is electrically isotropic. When stress is applied to silicon, the components in the resistivity matrix change.

$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ \rho_4 \\ \rho_5 \\ \rho_6 \end{bmatrix} = \begin{bmatrix} \rho \\ \rho \\ \rho \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta\rho_1 \\ \Delta\rho_2 \\ \Delta\rho_3 \\ \Delta\rho_4 \\ \Delta\rho_5 \\ \Delta\rho_6 \end{bmatrix} \tag{7}$$

Let us define the matrix of relative resistivity **d** as:

$$[d] = \frac{[\Delta\rho]}{\rho} \tag{8}$$

where $\Delta\rho$ is resistivity variation and ρ is unstrained resistivity. The electric field – current relation can be written in more general form as:

$$\frac{1}{\rho}\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} + \begin{bmatrix} d_1 & d_6 & d_5 \\ d_6 & d_2 & d_4 \\ d_5 & d_4 & d_3 \end{bmatrix} \begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix} \tag{9}$$

The relative resistivity is related to the stress by the piezoresistive effect. As the stress itself and resistivity are a second-rank tensors, the piezoresistive effect requires a four-rank tensor of piezoresistive coefficients for its full description. Due to symmetry conditions, this tensor is populated by only three non-zero components, as shown:

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} \pi_{11} & \pi_{12} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{11} & \pi_{12} & 0 & 0 & 0 \\ \pi_{12} & \pi_{12} & \pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \pi_{44} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} \qquad (10)$$

Piezoresistive tensor coefficients, p*ij* have units of Pa^{-1}. They may be either positive or negative and are sensitive to doping type, doping level and operating temperature. It is evident that p_{11} relates the resistivity in any direction to stress in the same direction, whereas p_{12} and p_{44} are cross-terms. Following table gives typical room-temperature values of piezoresistance coefficients and resistivity for p-type and n-type silicon:

Silicon	**Resistivity [? cm]**	p_{11} **[10^{-11} Pa^{-1}]**	p_{12} **[10^{-11} Pa^{-1}]**	p_{44} **[10^{-11} Pa^{-1}]**
p-type ($1.5.10^{15}/cm^3$)	7.8	6.8	-1.1	142.7
n-ype ($4.10^{14}/cm^3$)	11.7	-105.6	55.2	-14

Table 1.1 Piezoresistance coefficients and resistivity for p-type and n-type silicon.

Preceding equation was derived in the context of a coordinate system aligned to the (100) axes and is not always convenient to apply. A preferred representation is to express the fractional change in an arbitrarily oriented diffused resistor by:

$$\frac{\Delta R}{R} = \pi_L T_L + \pi_T T_T \qquad (11)$$

Where p_L and T_L are the piezoresistive coefficient and stress parallel to the direction of current flow in the resistor (i.e., parallel to its length), and p_T and T_T are the values in the transverse direction. These two orientations of piezoresistor relatively to the applied stress correspond to the most common situations in piezoresistive sensor devices and are shown on following figure.

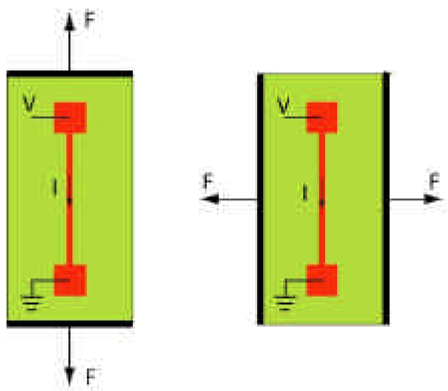

Figure 1.2 Longitudinal and transversal orientation of piezoresistors.

The piezoresistive coefficients referenced to the direction of the resistor may be obtained from those referenced to the (100) axes by using a transformation of the coordinate system. As most of the silicon micromachined devices if made of (100) wafers we will give in the following table the piezoresistive coefficients for two possible gauge orientations in this case.

Longitudinal Direction	π_L	**Transversal Direction**	π_T
[100]	π_{11}	[010]	π_{12}
[110]	$\frac{1}{2}(\pi_{11}+\pi_{12}+\pi_{44})$	$[1\bar{1}0]$	$\frac{1}{2}(\pi_{11}+\pi_{12}-\pi_{44})$

Table 1.2 Piezoresistive coefficients for two possible gauge orientations.

Polycrystalline silicon (polysilicon) based gauges can be used in some applications. Polysilicon is composed of many small single crystals or grains of silicon joined together by grain boundaries. Many polysilicon properties are similar to that of single crystal silicon; other characteristics are strongly influenced by the grain boundaries rather than by grains themselves. Piezoresistance coefficients for large grained polysilicon can approach 60-70% of single crystal silicon; however, for fine-grained, micromechanical polysilicon, p_L is less than 15 % of piezoresistance coefficient of single crystal silicon. Advantages of polysilicon and especially of its fine grained form are high degree of thickness uniformity, smooth surfaces and low defect densities, homogeneous, repeatable mechanical properties, and better line width control than with other, larger-grained films. Disadvantages of polysilicon include temperature coefficient of expansion mismatches with substrate that can cause unwanted stress, and the dependency of all physical properties on film morphology and processing history.

In order to calculate expected p coefficients for polysilicon, it is necessary to take into account the probability of orientations of single grains. If more precise estimation is demanded, the effect of grain boundaries must be included.

Currently there are two kinds of piezoresistive sensors made. Cantilever beams sensors are made for accelerometers while membrane sensors are manufactured to measure pressure and flow. We will discuss these two structural examples in order to show possible configurations of piezoresistors.

On cantilever beam sensors, a piezoresistor is built on the surface of the beam near its support in order to utilise stress concentration, as shown in the following figure.

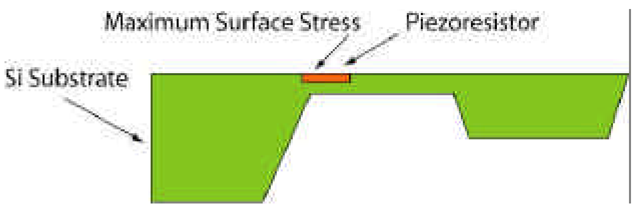

Figure 1.3 Cantilever beam sensor with piezoresistive gauge.

If the beam support can be considered as rigid in comparison to the beam, the maximum bending stress at the surface occurs right at the beam fixed end. If this condition is not valid, and beam flexural stiffness became comparable to the substrate stiffness, numerical modelling is required to determine the location of maximum stress. If we suppose the beam made by a bulk micromachining of (100) silicon wafer, its orientation will be along a [110] direction. In the same direction will appear the stress when the beam is bent. There are two possible orientations of piezoresistors as shown in top view in the Figure 1.4.

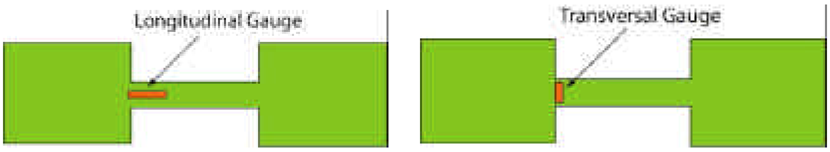

Figure 1.4 Longitudinal and transversal piezoresistors placements.

With the values from Table 1.1 and with the expressions from Table 1.2 we obtain the longitudinal and transversal piezoresistive coefficients as shown in Table 1.3.

Silicon	p_L **[10^{-11} Pa^{-1}]**	p_T **[10^{-11} Pa^{-1}]**
p-type	74.2	-68.5
n-ype	-32.2	-18.2

Table 1.3 Longitudinal and transversal piezoresistance coefficients.

We can see that the p-type piezoresistors have larger sensitivity and that longitudinal and transversal coefficients have opposite signs and almost equal magnitudes. This makes p-type piezoresistors well suited for bridge applications. The choice between two orientations of piezoresistors may be done based on system requirements. Transversely oriented piezoresistor has the potential for largest response because, if it can be placed exactly at the place of maximal bending stress, the entire resistor will experience it. The disadvantage of this orientation is that it is very susceptible to manufacturing variations. Longitudinaly oriented piezoresistor must extend over some length along the cantilever and may also extend onto the support. This results to some loss of sensitivity because of the fact that not every part of the resistor experiences the maximum stress. Longitudinal orientation thus requires less costly manufacturing procedure. The final position of the piezoresistor on the membrane must take into account expected fabrication reproducibility. A mixed solution, using two gauges of both orientations can be also considered. In this case the opposite sign of piezoresistive coefficients will give higher voltage at the output of the Wheatstone bridge arrangement.

It is convenient to form the longitudinal piezoresistance of several segments in order to achieve more uniform stress distribution over its length as shown in Figure 1.5. Required resistance value is obtained by the choice of number of segments.

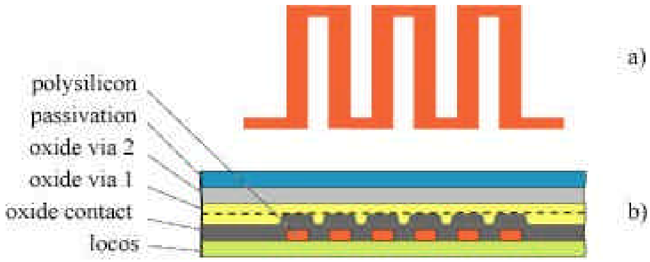

Figure 1.5 Piezoresistor composed of several segments, a) top view b) cross section.

One of possible dispositions of piezoresistors on the membrane is shown in the Figure1.6. Membrane sensors are usually designed as thin single crystal silicon plates supported on the borders by a thick mass of silicon substrate. In order to maximize sensitivity, it is common to use several piezoresistors in a bridge circuit. Usually a piezoresistor is built in the proximity of the edge of the membrane to utilize stress concentration due to the deformation by externally applied load. Unlike cantilever beam where only one stress direction can be detected, in the case of membrane, stresses in two perpendicular directions are experienced. If one of piezoresistors experience a longitudinal stress T_L, then it must simultaneously experience a transverse stress $T_T = \nu\, T_L$. As a result, the total change in resistance, assuming uniform stress over the entire resistor, would be

$$\frac{\Delta R}{R} = (\pi_L + \upsilon\, \pi_T) T_L \qquad (12)$$

Taking into account the Hooke's relation between stress and strain, we can write the preceding relation in the form:

$$\frac{\Delta R}{R} = (\pi_L + \upsilon\, \pi_T) E\, S_L = K\, S_L \qquad (13)$$

where K is the gauge factor. Values of piezoresistive coefficients and of Poisson ratio and thus of gauge factor depend on the material and also on given orientation. We can use p-type piezoresistors that have, as we have seen in Table 1.3, high values of piezoresistive coefficients.

Figure 1.6 shows the configuration of four p-type piezoresistors, theirs positions on the membrane and also the membrane orientation relatively to the plane of the wafer. It must be remembered that to use a p-type piezoresistor, one must diffuse it into an n-type substrate to achieve junction isolation. Alternatively, to isolate piezoresistors from each other and from substrate, polycrystalline silicon surface-micromachined structures can be used.

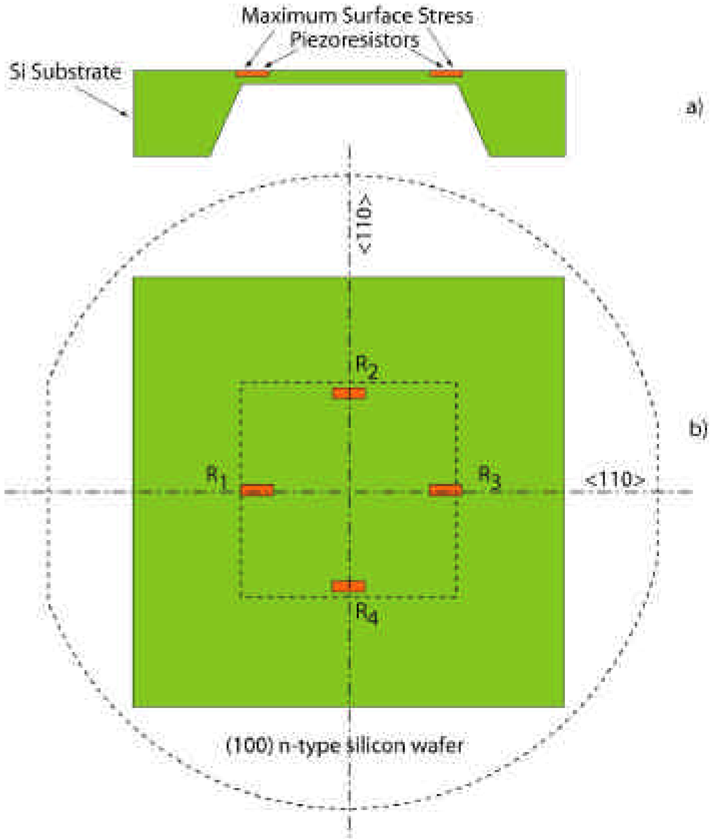

Figure 1.6 Piezoresistors on a membrane, a) cross section, b) top view.

To convert the resistance change to a voltage signal, the Wheatstone-bridge configuration shown in Figure 1.7 is used. Equally positioned resistors form opposite arms of the bridge so that under applied pressure, the left and right output nodes of the bridge deviate from their zero-pressure voltage with opposite signs.

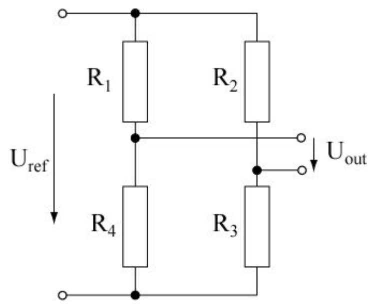

Figure 1.7 Wheatstone-bridge configuration.

We can write for the bridge output voltage, U_{out} and for its change dU_{out}:

$$U_{out} = U_{ref}\left(\frac{R_1}{R_1+R_4}-\frac{R_2}{R_2+R_3}\right) \qquad (14)$$

$$dU_{out} = \sum_{i=1}^{4}\frac{\partial U}{\partial R_i}dR_i = U_{ref}\left(\frac{R_4 dR_1 - R_1 dR_4}{(R_1+R_4)^2}-\frac{R_3 dR_2 - R_2 dR_3}{(R_2+R_3)^2}\right) \qquad (15)$$

If all resistors have the same resistance, R_0 for the unloaded case, due to their dimensioning, and if the absolute value of all resistance changes is equal to dR, then the simplified expression can be written:

$$dU_{out} = U_{ref}\frac{dR}{R_0} \tag{16}$$

We will take, for a numerical example, the configuration from Figure 1.6 and the values from Table 1.3. The resistors R_1 and R_3 compared to R_2 and R_4 experience stresses rotated 90°. We should take into account that the longitudinal stress on R_1 and R_3 is the transverse stress at R_2 and R_4 and vice versa. We can write for the change of resistances:

$$\frac{\Delta R_1}{R_1} = (69.8\,10^{-11})T_L\ ,\quad \frac{\Delta R_2}{R_2} = (-63.7\,10^{-11})T_L \tag{17}$$

Supposing that unloaded values of all resistances are the same and equal to R_0, we can write:

$$R_1 = R_3 = (1+\alpha_1)R_0\ ,\quad R_2 = R_4 = (1-\alpha_2)R_0 \tag{18}$$

where $\alpha 1$ and $\alpha 2$ represent the product of the effective piezoresistive coefficient and the stress. For the bridge output voltage the following expression holds:

$$U_{out} = U_{ref}\frac{R_1R_3 - R_2R_4}{(R_1+R_2)(R_3+R_4)} \approx \frac{\alpha_1+\alpha_2}{2(1+\alpha_1-\alpha_2)} \tag{19}$$

Since $\alpha 1$ and $\alpha 2$ are typically small, and differ from each other by only 10%, this bridge gives an optimally large output without a large nonlinearity.

The Wheatstone-bridge configuration has some distinct advantages. It converts the resistance change directly to a voltage signal. The output voltage is independent of the absolute value of the piezoresistors, but is determined by the relative resistance change and the bridge voltage. In the ideal case, the total resistance of bridge is independent of pressure since the resistance changes cancel one another. Moreover, in the case of a perfectly balanced bridge, temperature influences and other common-mode effects are compensated by an equal variation of all piezoresistors.

1.2 Related Reading

[MID 89] MIDDELHOEK, S., AUDET, S. A., *Silicon sensors*, Academic Press, 1989.

[RUF 02a] RUFER L., *La modélisation des microsystèmes électromécaniques*. in Mir, S. (Ed.), *Conception des microsystèmes sur silicium*. Traité EGEM, Hermes Science Publications, pages 101-128, 2002.

[RUF 02b] RUFER L., *Les microsystèmes électromécaniques*. in Mir, S. (Ed.), *Les applications des microsystèmes sur silicium*. Traité EGEM, Hermes Science Publications, pages 19-64, 2002.

[SEN 01] SENTURIA S. D., *Microsystem Design*, Kluwer Academic Publishers, 2001.